IEMS 490/ CS 496: Selected Topics in Modern Discrete Probability

Instructor Julia Gaudio
julia.gaudio@northwestern.edu

Course Topics This is a graduate-level course focused on techniques and models in modern discrete probability. Topics include: the first and second moment methods, Chernoff bounds and large deviations, martingales, concentration inequalities, branching processes, percolation, and Markov chains. Examples will be drawn from random structure and algorithm applications. The goal of the course is to equip students to carry out their own research using the toolkit of discrete probability.

Prerequisites Strong background in probability and stochastic processes, at the level of IEMS 460-1, or permission of the instructor.

Topics

1. First and Second Moment Methods (3 lectures)
    - Markov inequality
    - Chebyshev inequality
    - Paley–Zygmund inequality
    - Applications: the Erdős–Rényi random graph model (connectivity threshold, subgraph counts); percolation on $\mathbb{Z}^d$ and on the binary tree
2. Chernoff Bounds and Large Deviations (3 lectures)
3. Martingales and Concentration (4 lectures)
    - Doob's Upcrossing Inequality
    - Martingale Convergence Theorem
    - Optional Stopping Theorem

- Azuma–Hoeffding and McDiarmid inequalities; applications to graph coloring, balls and bins, longest common subsequence

4. Branching Processes (2 lectures)
    - Galton–Watson trees
    - Percolation on the d-ary tree
    - Giant component in sparse Erdős–Rényi random graphs
5. Markov Chains (6 lectures)
    - Mixing time
    - Coupling
    - Glauber dynamics
    - Spectral methods
    - Path coupling
    - Lower bounds on mixing time
    - MCMC
6. Additional Techniques (4 lectures)
    - Correlation inequalities (e.g. FKG bound)
    - Janson's inequality; application to triangles in Erdős–Rényi random graphs, chromatic number
    - Efron–Stein inequality
    - Talagrand's inequality

Sources

- Scribe lecture notes from the MIT course 15.070/6.265 as taught by Yury Polyanskiy and Guy Bresler in 2017.
- Notes from the MIT course 15.070/6.265 (2018 and 2021) and the Harvard course MATH 295 (2018), as taught by David Gamarnik.
- Notes from Modern Discrete Probability: An Essential Toolkit (University of Wisconsin Madison) as taught by Sebastien Roch.
- Alon, N., and Spencer, J. H. (2016). *The Probabilistic Method*. John Wiley & Sons.
- Boucheron, S., Lugosi, G., and Massart, P. (2013). *Concentration Inequalities: A Nonasymptotic Theory of Independence.* Oxford University Press.

- Levin, D. A., and Peres, Y. (2017). *Markov Chains and Mixing Times* (Vol. 107). American Mathematical Soc..
- Janson, S., Rucinski, A., and Luczak, T. (2011). *Random Graphs*. John Wiley & Sons.
- Mitzenmacher, M., Upfal, E. (2017). *Probability and computing: Randomization and probabilistic techniques in algorithms and data analysis*. Cambridge University Press.

Schedule

- Class: Tuesdays and Thursdays 12:30-1:50
- Office Hours: TBD

Grading 50% for problem sets, 20% for the take-home midterm exam, 30% for the take-home final exam.

Academic Integrity Students in this course are required to comply with the policies found in the booklet, "Academic Integrity at Northwestern University: A Basic Guide". For details regarding academic integrity at Northwestern or to download the guide, visit: https://www.northwestern.edu/provost/policies-procedures/academic-integrity/index.html.

We will follow the McCormick academic integrity policy. Violations include:

- Looking at another student's exam.
- Accessing prior course materials.
- Sharing course materials with future students.

Any suspected violations will be reported to the Dean's office. Violations will result in an F grade.

If you discuss homework problems with other students, you must write their names at the top of your submission. All solutions must be written independently. No collaboration is permitted on the midterm and final.

Late Policy Homework is due by 5:00 pm on Wednesdays. It must be submitted on Crowdmark.

The late penalty is

- 10% if up to 24 hours late
- 20% if 24 to 48 hours late
- No credit if over 48 hours late.

Health and Safety *Need to update this closer to the fall*

Accessibility Northwestern University is committed to providing the most accessible learning environment as possible for students with disabilities. Should you anticipate or experience disability-related barriers in the academic setting, please contact AccessibleNU to move forward with the university's established accommodation process (email: accessiblenu@northwestern.edu; phone: 847-467-5530). If you already have established accommodations with AccessibleNU, please let me know as soon as possible, preferably within the first two weeks of the term, so we can work together to implement your disability accommodations. Disability information, including academic accommodations, is confidential under the Family Educational Rights and Privacy Act.

Wellness and Mental Health Northwestern University is committed to supporting the wellness of our students. Student Affairs has multiple resources to support student wellness and mental health. If you are feeling distressed or overwhelmed, please reach out for help. Students can access confidential resources through the Counseling and Psychological Services (CAPS), Religious and Spiritual Life (RSL) and the Center for Awareness, Response and Education (CARE). Additional information on all of the resources mentioned above can be found here:

https://www.northwestern.edu/counseling/
https://www.northwestern.edu/religious-life/
https://www.northwestern.edu/care/

Other policies

- If you believe there was a grading error, you may submit a re-grade request. Re-grade requests must be submitted within one week of the assignment being returned. The request must include a justification.
- Unauthorized student recording of classroom or other academic activities (including advising sessions or office hours) is prohibited. Unauthorized recording is unethical and may also be a violation of University policy and state law. Students requesting the use of assistive technology as an accommodation should contact AccessibleNU. Unauthorized use of classroom recordings – including distributing or posting them – is also prohibited. Under the University's Copyright Policy, faculty own the copyright to instructional materials – including those resources created specifically for the purposes of instruction, such as syllabi, lectures and lecture notes, and presentations. Students cannot copy, reproduce, display, or distribute these materials. Students who engage in unauthorized recording, unauthorized use of a recording, or unauthorized distribution of instructional

materials will be referred to the appropriate University office for follow-up.